Fair Curves and Surfaces

Theses of the Ph.D. Dissertation

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1 Introduction

Creating aesthetically pleasing, *fair* curves and surfaces is a principal issue of Computer Aided Geometric Design (CAGD). It is especially so for automobiles and other consumer goods, such as household appliances, where sales largely depend on the appearance of the products. Using today's technology, professional designers still have to spend many days manually adjusting control points in order to create Class A surfaces with smooth connections.

Fairness is an elusive concept. There is no exact mathematical definition, and different applications may have different requirements. Still, researchers agree that evenly distributed curvature is favorable [Far02, RR94]. It is not sufficient, though, just to smooth the surface: we have to preserve the highly curved features of the original model. Locality and deviation control are thus important factors in fairing algorithms.

Most real-life models, where appearance counts, contain smooth edges, usually defined by fillet surfaces and corner patches. These need to be continuously joined to the primary surfaces, so besides individual fairing, care should be taken to enhance the connections between surfaces as well. In this context, "smooth connections" usually mean G^2 continuity. G-continuity is less strict than C-continuity: it is sufficient that there exists a parameterization of the surfaces such that C-continuity applies [Far02]. Informally speaking, this means that for G^1 connections the two surfaces share a common tangent plane at every point of the boundary curve, while for G^2 connections they also have matching surface curvatures. These are often referred to as *tangent continuity* and *curvature continuity*, respectively.

Fair curves and surfaces are crucial in many practical applications. One of these is Digital Shape Reconstruction (DSR), which deals with the creation of geometric models based on measured data. Its workflow comprises triangulation, segmentation, classification, surface fitting and surface improvement [VM02]. It is a very complex procedure, where even slight measurement errors can cause significantly decreased quality. This is why fairing methods are indispensable in DSR. So-called *variational* methods can be integrated into the surface approximation process, playing an important part in the surface fitting phase. In contrast, *post-processing* fairing algorithms are applied at the final improvement stage.

Another fundamental application of fairing is in surface design. There are various ways to build a model; one approach is to first create a curve network representing the edges and feature lines of the actual object. The curves may come from several sources, like traditional blueprints, 2D sketches, or directly by some GUI interface. These curves need to be smooth, in order to be able to fit high-quality surfaces onto them at a later stage. In curvenet-based design it is also crucial to adopt the most suitable types of surfaces. *Transfinite interpolation surfaces* are particularly suitable for this purpose, since they are defined solely by the boundary curves and their cross-derivatives.

2 Research Goals and Methodology

This research is aimed at developing new, automatic or semi-automatic algorithms for creating fair geometries. Part of the research deals with post-processing methods that apply fairing to already defined free-form curves and surfaces. This is usually the case in the shape reconstruction process. In this context, locality and preserving the features of the original geometry are key concerns. The new algorithms should be able to cope with these problems while smoothing a curve or surface. Parameterization-independence is also deemed to be a desirable property.

When a complete object composed of many connected surfaces needs to be faired, not only the smoothing of the individual surfaces is needed, but the hierarchy of surfaces must be taken into account as well. The fairing of dependent connection surfaces, such as fillets or corner patches, thus poses another problem. According to functional decomposition described in [VM02], a complex CAD model can be broken down into a set of surfaces with continuity constraints. Typically there is a hierarchy comprising (i) primary surfaces (ii) connecting surfaces, such as fillets, and (iii) corner patches, see Fig. 1.

Therefore, fairing should also be performed accordingly, providing continuity constraints from the previous phases. A general observation is that while primary surfaces are relatively large and are supposed to preserve the

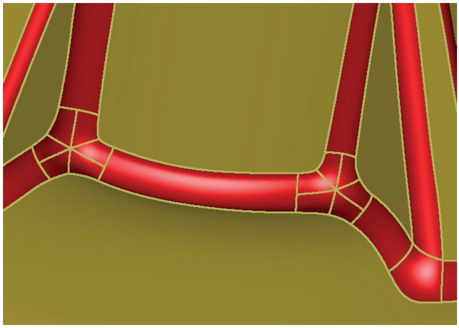

Figure 1: Primaries, connections, corner patches.

original design intent, fillets and corner patches, being much smaller (and the related measured data points less accurate), are more lenient about deviation in favor of ensuring continuity and fairness.

Another part of this research deals with the construction of fair multi-sided surface patches using transfinite surface interpolation, enhancing existing formulations as well as creating new ones. This contains several separate subproblems, such as (i) constructing interpolants, (ii) finding suitable domain polygons, (iii) defining parameterizations that map the domains of the interpolants onto the n-sided polygonal domain, and (iv) designing blending functions to combine the interpolant surfaces.

The work started with a thorough review of the literature [1, 6], and the development of a reliable testbed environment, where prototypes for existing methods and new ideas could easily be implemented. This helped realizing the shortcomings of conventional fairing algorithms: the inefficiency of curvature variation-based methods and the lack of fairing options for dependent and/or multi-sided surface patches. Then new ideas were put into practice and tested on several models, coming from both artificial and real measured data. For the new surface representations, rigorous mathematical proofs were given, as well. The results were presented at international conferences and were published in critically acclaimed journals.

3 Previous Work

There is no such thing as the best fairness measure. Different applications require different approaches, and this is the reason why an abundance of measures coexists in the literature. One well-known method has its roots in 18th century shipbuilding technology, where in order to draw a smooth curve, metal weights were placed at the interpolation points and a flexible spline was spanned between them. The resulting curve minimizes the squared curvature. As computing the curvature can be difficult, it is often replaced by (parameter-dependent) second derivatives. This is a questionable — though widely used — practice, since the fairing process may have unexpected results, when the curve's parameterization is substantially different from the arc-length parameterization.

One classical definition of fairness by Farin and Sapidis [Far02] states, that "A curve is fair if its curvature plot is continuous and consists of only a few monotone pieces." In other words, sudden changes in the curvature and inflections are considered unfavorable. The energy associated with Minimum Variation Curves (MVC) and Surfaces (MVS), proposed by Moreton and Séquin [MS94], respects this assumption, by minimizing the squared variation of curvature. Note, however, that this measure is very computation-intensive, especially for surfaces.

Another way to look at these energies is to find a "perfect" curve or surface (with zero energy): for the spline energy, this would be a line, since it penalizes curvature, but the MVC energy uses the variation of curvature, so the zero-energy curve would be a circular arc. Roulier and Rando [RR94] give a very comprehensive, detailed review on these and other fairness measures and their effects.

One of the simplest, but popular curve fairing algorithm is the *knot removal and reinsertion* (KRR), originally conceived by Kjellander and later optimized by Farin et al. [Far02]. This method uses the fact that (i) removing a knot from a B-spline curve's knot vector makes it more smooth (but of course, the details around the knot are lost), and that (ii) a new knot can be added without changing the shape of the curve. The idea is to remove a knot and then reinsert it, resulting in a smoother curve with the same degree of freedom. The removal step can be solved by moving only one control point. The iteration of this process, combined with some heuristics to choose the next knot based on a fairing measure, is a very efficient algorithm both in computational time and in quality. Unfortunately, its extension to surfaces is much less usable, since it can only guarantee fairing in one parametric direction, which is not satisfactory in real-life applications.

Bicubic Coons patches [Coo67, Far02] are parametric surfaces over four boundary curves and the corresponding cross-derivative functions. These generally have nice surface quality with even curvature distribution. Transfinite surface interpolation deals with multi-sided variants of Coons patches. Charrot and Gregory [CG84] proposed an n-sided patch based on *corner interpolants*, i.e., surfaces that interpolate two adjacent boundary curves. The interpolants are then composed using special blending functions.

Kato [Kat00] and several other authors used linear side interpolants or *ribbons* to define a different patch. While ribbons seem to be a more intuitive approach, the blending functions of the patch have the drawback of being singular in the corners. The above surfaces are in some ways similar to the Coons patch, but they are not generalizations in a strict sense, as they do not share the same structure.

4 Research Results

1st thesis. Fairing based on Target Curvature [1, 2, 3]

1.1. I have elaborated a new technique for evaluating the quality of curves and surfaces. In this approach, the deviation from a smooth target curvature is computed, thus it optimizes curvature variation; at the same time it is cheap to compute.

1.2. I have developed a curve-fairing method based on the above fairness concept. The algorithm is very efficient, due to the use of numerical integration, and provides high-quality results. Deviation control and locality ensures that the original features of the curve are preserved.

1.3. I have generalized the algorithm for surface fairing in two different ways. The isocurve-based extension is faster, while the direct extension yields better quality. Both methods are local.

1.4. I have also developed another surface fairing operation, applying Greiner's method of computing an approximate Hessian matrix [Gre96] in order to detemine target curvature. This variant boasts speed and robustness, while providing almost the same improvement in quality.

Figure 2: Isophotes of a car body panel faired by the curvature approximation method (1.4)

2nd thesis. Hierarchical Fairing with Constraints [4, 5]

2.1. I have created a master–slave algorithm to smoothly connect two adjacent surfaces with curvature continuity (G^2) in a numerical sense. The process also ensures that the control net of the modified surface is only minimally altered.

2.2. I have proposed a hierarchical workflow for fairing complex models with continuity constraints in accordance with the functional decomposition paradigm used in Digital Shape Reconstruction. In the course of fairing primary surfaces, emphasis is placed on preserving the original shape features, while for connecting surfaces, such as fillets and vertex blends, the algorithm is tuned toward fairness and continuity.

2.3. I have developed a novel method for four-sided patches that comprises alternating steps of satisfying continuity constraints and fairing the surface interior. For the fairing steps, various local methods can be used, including the ones introduced in 1.3.–1.4.

2.4. I have extended the above method for n-sided patches composed of quadrilaterals. In the first step the full patch needs to be faired by applying either discrete fairing or a genuine n-patch formulation. This is followed by constrained fitting of the surfaces. This approach guarantees that the low-quality regions of the patch are eliminated along the internal subdividing boundaries.

3rd thesis. Transfinite n-sided Surfaces [6, 7]

3.1. I have enhanced former classical schemes by applying non-regular domain polygons. These enable parameterizations that adapt better to three-dimensional boundary conditions (including lengths and angles), and thus make it possible to get rid of undesirable surface artifacts. Three new algorithms have been elaborated to define domain polygons.

3.2. I have developed a new parameterization scheme, called *central line sweep*, that produces well-oriented ribbon mappings and overcomes problems of traditional methods over non-regular polygonal domains.

3.3. I have proposed a new approach for transfinite surface interpolation by applying curved ribbons instead of the most frequently used linear

Figure 3: Fairing an X-node (junction of four fillets)

ribbon interpolants. This method yields more predictable shapes, in particular for highly curved boundary configurations.

3.4. I have introduced a new surface representation that can be considered as the true multi-sided generalization of the classical (four-sided) Coons patch. I have also designed new parameterization schemes (*interlinked*, *parabolic* and *bi-quadratic* methods) that satisfy the more strict differential requirements of this new n-sided patch.

3.5. I have extended the previous formula to combine curved ribbons in a natural manner. I have also proved that the parameterization requirements can be relaxed, when it is sufficient to reproduce only the normal vectors of the ribbons along the boundaries instead of the exact cross-tangents. This G^1 patch also provides natural shapes and is well-suited for many applications.

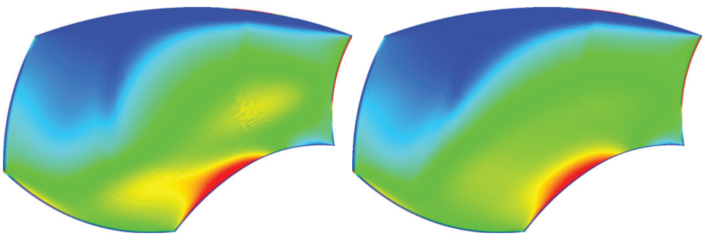

Figure 4: Mean map comparison of a surface applying the generalized Coons patch (3.4, left) and the composite ribbon patch (3.5, right)

5 Applications

Some results of the first thesis were incorporated into a Small Business Innovation Research grant "Creating functionally decomposed surface models from measured data" (SBIR #0450230) of the American National Science Foundation, when I was working with the international development team of Geomagic, Inc.

The third thesis, as a whole, was integrated into the Sketches prototype modeling system developed by ShapEx Ltd. This research and development effort was supported by a cooperation with the King Abdullah University of Science and Technology. The sketch-based technology in this software is illustrated in Fig. 5.

The research was also part of the "Development of quality-oriented and harmonized R+D+I strategy and functional model at the Budapest University of Technology and Economics" project (UMFT-TÁMOP-4.2.1/B-09/1/KMR-2010-0002).

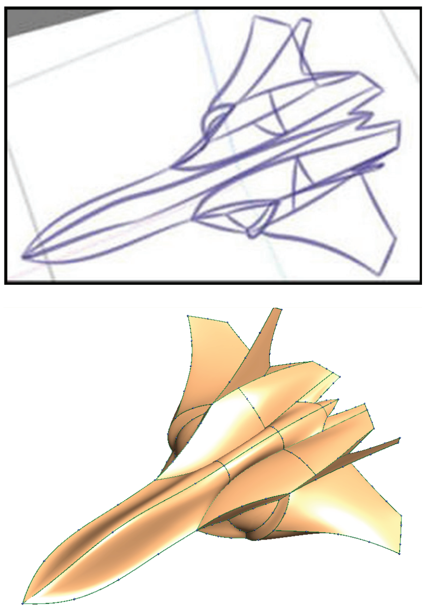

Figure 5: "Jetfighter" model — original 3D sketch (top) and the generated surface model (bottom)

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Related Publications

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